TRINITY COLLEGE

Year 12 Methods Units 3,4
Test 2 2021

Section 1 Calculator Free

Area, Fundamental Theorem, Exponential Function

STUDENT'S NAME _____

DATE: Thursday 25 March **TIME:** 25 minutes **MARKS**: 25

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

1. (4 marks)

Determine $\frac{d}{dx}$ for each of the following.

(a) $e^{1-x}(x^e - 8)$ [2]

(b) $\int_{-1}^{x} \frac{e^{\pi} - e^{t+1}}{\sqrt{1+t}}\, dt$ [2]

2. (11 marks)

(a) Determine each of the following.

(i) $\int 3xe^{x^2-6}dx$ [2]

(ii) $\int \frac{8e^{2x}+e^{-x+1}}{e^{-x}}dx$ [3]

(b) Given $\frac{dP}{dt}=e^{4-2t}$ determine an expression for P if $P=\frac{e^2}{2}$ when $t=1$ [3]

(c) Evaluate $\int_0^1 \frac{d}{dx}\left(\frac{x^3}{x^2+1}\right)dx$ [3]

3. (6 marks)

(a) Determine $\frac{d}{dx} xe^{2x}$ [2]

Hence or otherwise evaluate exactly

(b) $\int_0^1 2xe^{2x} dx$ [4]

4. (4 marks)

Determine and classify all stationary points of the curve $y = x^2 e^x$.

TRINITY COLLEGE

Year 12 Methods Units 3,4
Test 2 2021

Section 2 Calculator Assumed

Area, Fundamental Theorem, Exponential Function

STUDENT'S NAME _____

DATE: Thursday 25 March **TIME:** 25 minutes **MARKS**: 28

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

---

5. (4 marks)

Determine the area enclosed by $x = 3$, $y = 5$ and $y = \dfrac{2}{x^2 - 1}$.

6. (6 marks)

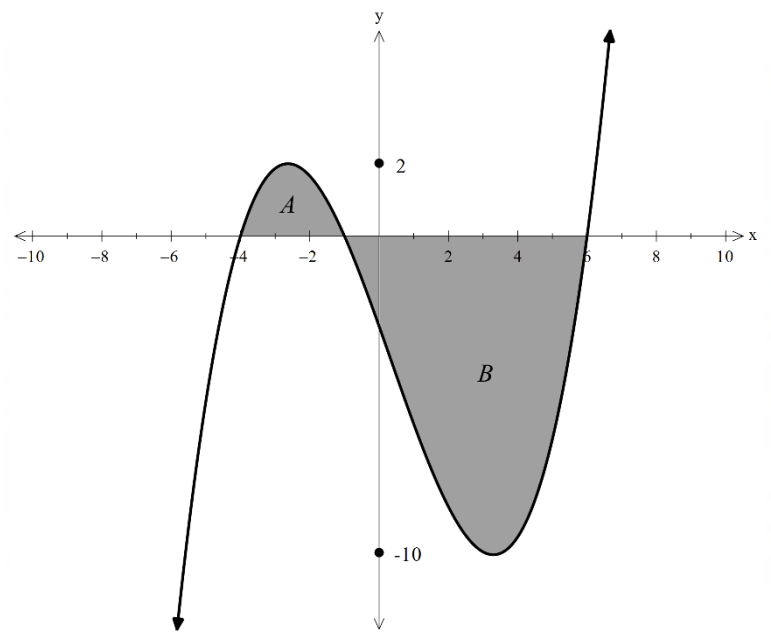
Scientists study a population of mice over a ten week period and conclude that the population is increasing at a rate given by $R'(t) = 28e^{0.15t}$ where t is the number of weeks since the study began and an initial population of 30 mice.

(a) What is the change in the population in the seventh week of the study? [2]

(b) What is the average weekly increase in mice over the ten week period? [2]

(c) How long does it take for the mice population to reach 150? [2]

7. (10 marks)



In the diagram above showing the graph of $y = f(x)$, the shaded region *A* has an area of 5 square units. Shaded region *B* has an area of 30 square units.

Using the information above, determine

(a) $\int_{-4}^{6} 2f(x)dx$ [2]

(b) $\int_{-4}^{-1} (f(x)+2)dx$ [3]

(c) $\int_{-3}^{2} f(2x+2)dx$ [3]

(d) $\int_{-1}^{6} f'(x)dx$ [2]

8. (4 marks)

A curve for which $\frac{dy}{dx} = -e^{kx}$, where k is a constant, is such that the tangent at $(1, -e^3)$ passes through the origin.

(a) Determine the gradient of the tangent. [1]

(b) Determine the equation of the curve. [3]

9. (4 marks)

A continuous function $f(x)$ is increasing on the interval $0 < x < 2$ and decreasing on the interval $2 < x < 6$. Some of its values are given in the table below.

x	0	1	2	3	4	5	6
$f(x)$	5	16	27	23	16	0	-12

The function $F(x)$ is defined for $0 \le x \le 6$ by $F(x) = \int_0^x f(t)dt$.

(a) At which value of x in the interval $0 \le x \le 6$ is $F(x)$ the greatest? Justify your answer.

[2]

(b) At which value of x in the interval $0 \le x \le 6$ is $F'(x)$ the greatest? Justify your answer.

[2]